$$\hat{A}\hat{B}-\hat{B}\hat{A}=[\hat{A},\hat{B}]$$

$$[\hat{B},\hat{A}]=-[\hat{A},\hat{B}]$$

$$[\hat{A},\hat{B}+\hat{C}]=[\hat{A},\hat{B}]+[\hat{A},\hat{C}]$$

$$[\hat{A},\hat{B}\hat{C}]=\hat{B}[\hat{A},\hat{C}]+[\hat{A},\hat{B}]\hat{C}$$

$$[\hat{A}\hat{B},\hat{C}]=\hat{A}[\hat{B},\hat{C}]+[\hat{A},\hat{C}]\hat{B}$$

$$[\hat{J}_x,\hat{J}_y]=i\hbar\hat{J}_z$$

$$[\hat{J}_z,\hat{J}_x]=i\hbar\hat{J}_y$$

$$[\hat{J}_y,\hat{J}_z]=i\hbar\hat{J}_x$$

Commuting Hermitian Operators

$$[\hat{A}, \hat{B}] = 0 \Rightarrow \hat{A}\hat{B} - \hat{B}\hat{A} = 0 \Rightarrow \hat{A}\hat{B} = \hat{B}\hat{A}$$

Non-degenerate:

$$\hat{A}|a_1\rangle = a_1|a_1\rangle$$
$$\hat{A}|a_2\rangle = a_2|a_2\rangle$$
$$\hat{A}|a_3\rangle = a_3|a_3\rangle$$

$$\hat{A}\big(\underbrace{\hat{B}|a_1\rangle}_{b|a_1\rangle}\big) = \hat{B}\hat{A}|a_1\rangle = \hat{B}\,a_1|a_1\rangle = a_1\big(\underbrace{\hat{B}|a_1\rangle}_{b|a_1\rangle}\big)$$

$$\hat{B}|a_1\rangle = b|a_1\rangle$$

$$[\hat{A},\hat{B}]=0 \Rightarrow \hat{A}\hat{B}-\hat{B}\hat{A}=0 \Rightarrow \hat{A}\hat{B}=\hat{B}\hat{A}$$

Degenerate:

$$\hat{A}|a_1\rangle = a|a_1\rangle$$

$$\hat{A}|a_2\rangle = a|a_2\rangle$$

$$\hat{A}|a_3\rangle = a|a_3\rangle$$

$$\hat{A}\left(\hat{B}|a_1\rangle\right)=\hat{B}\hat{A}|a_1\rangle = \hat{B}a|a_1\rangle = a\left(\underbrace{\hat{B}|a_1\rangle}\right)$$

$b|a_1\rangle$?

$b|a_2\rangle$?

$b|a_3\rangle$?

$$\hat{B}|a_1\rangle \neq b|a_1\rangle$$

$$\hat{J}^2 = \hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2$$

$$[\hat{J}_z, \hat{J}^2] = 0$$

$$[\hat{J}_x, \hat{J}^2] = 0$$

$$[\hat{J}_y, \hat{J}^2] = 0$$

$$[\hat{J}_x, \hat{J}_y] = i\hbar \hat{J}_z \neq 0$$

$|\lambda, m\rangle$ shared eigenstate of $\hat{J}_z$ and $\hat{J}^2$

$$\hat{J}^2 |\lambda, m\rangle = \lambda \hbar^2 |\lambda, m\rangle$$

$$\hat{J}_z |\lambda, m\rangle = m\hbar |\lambda, m\rangle$$

$$\hat{J}_+ = \hat{J}_x + i\hat{J}_y \qquad \hat{J}_+^{\dagger} = \hat{J}_x^{\dagger} - i\hat{J}_y^{\dagger} = \hat{J}_x - i\hat{J}_y = \hat{J}_-$$

$$\hat{J}_- = \hat{J}_x - i\hat{J}_y \qquad \hat{J}_-^{\dagger} = \hat{J}_+$$

$$[\hat{J}_x, \hat{J}_y] = i\hbar\hat{J}_z$$
$$[\hat{J}_z, \hat{J}_x] = i\hbar\hat{J}_y$$
$$[\hat{J}_y, \hat{J}_z] = i\hbar\hat{J}_x$$

$$[\hat{J}_z, \hat{J}_+] = [\hat{J}_z, \hat{J}_x + i\hat{J}_y] = \underbrace{[\hat{J}_z, \hat{J}_x]}_{i\hbar\hat{J}_y} + i\underbrace{[\hat{J}_z, \hat{J}_y]}_{-i\hbar\hat{J}_x} = i\hbar\hat{J}_y + \hbar\hat{J}_x = \hbar\hat{J}_+$$

$$[\hat{J}_z, \hat{J}_-] = [\hat{J}_z, \hat{J}_x - i\hat{J}_y] = \underbrace{[\hat{J}_z, \hat{J}_x]}_{i\hbar\hat{J}_y} - i\underbrace{[\hat{J}_z, \hat{J}_y]}_{-i\hbar\hat{J}_x} = i\hbar\hat{J}_y - \hbar\hat{J}_x = -\hbar\hat{J}_-$$

$$[\hat{J}_z, \hat{J}_\pm] = \pm\hbar\hat{J}_\pm$$

$$[\hat{J}_z, \hat{J}_\pm] = \pm\hbar \hat{J}_\pm \qquad \hat{J}_z|\lambda, m\rangle = m\hbar|\lambda, m\rangle \qquad \hat{J}^2|\lambda, m\rangle = \lambda\hbar^2|\lambda, m\rangle$$

$$\hat{J}_z\hat{J}_+ - \hat{J}_+\hat{J}_z = \hbar\hat{J}_+ \quad \Rightarrow \quad \hat{J}_z\hat{J}_+ = \hat{J}_+\hat{J}_z + \hbar\hat{J}_+$$

$$\hat{J}_z\left(\hat{J}_+|\lambda, m\rangle\right) = (\hat{J}_+\hat{J}_z + \hbar\hat{J}_+)|\lambda, m\rangle = \left(\hat{J}_+ m\hbar + \hbar\hat{J}_+\right)|\lambda, m\rangle = (m+1)\hbar\left(\underbrace{\hat{J}_+|\lambda, m\rangle}_{c_+\hbar|\lambda, m+1\rangle}\right)$$

$$\hat{J}_z\left(\hat{J}_+|\lambda, m\rangle\right) = (m+1)\hbar\left(\hat{J}_+|\lambda, m\rangle\right) \quad \Rightarrow \quad \hat{J}_+|\lambda, m\rangle = c_+\hbar|\lambda, m+1\rangle$$

$$\hat{J}_z\hat{J}_- = \hat{J}_-\hat{J}_z - \hbar\hat{J}_-$$

$$\hat{J}_z\left(\hat{J}_-|\lambda, m\rangle\right) = (\hat{J}_-\hat{J}_z - \hbar\hat{J}_-)|\lambda, m\rangle = \left(\hat{J}_- m\hbar - \hbar\hat{J}_-\right)|\lambda, m\rangle = (m-1)\hbar\left(\underbrace{\hat{J}_-|\lambda, m\rangle}_{c_-\hbar|\lambda, m-1\rangle}\right)$$

$$\hat{J}_z\left(\hat{J}_-|\lambda, m\rangle\right) = (m-1)\hbar\left(\hat{J}_-|\lambda, m\rangle\right) \quad \Rightarrow \quad \hat{J}_-|\lambda, m\rangle = c_-\hbar|\lambda, m-1\rangle$$

$$\hat{J}_-\hat{J}_+ = (\hat{J}_x - i\hat{J}_y)(\hat{J}_x + i\hat{J}_y) = \hat{J}_x^2 + i\hat{J}_x\hat{J}_y - i\hat{J}_y\hat{J}_x + \hat{J}_y^2$$

$$= \hat{J}_x^2 + \hat{J}_y^2 + i(\hat{J}_x\hat{J}_y - \hat{J}_y\hat{J}_x) = \hat{J}^2 - \hat{J}_z^2 + i[\hat{J}_x, \hat{J}_y] = \hat{J}^2 - \hat{J}_z^2 - i\hbar\hat{J}_z$$

$$\hat{J}_-\hat{J}_+ = \hat{J}^2 - \hat{J}_z^2 - \hbar\hat{J}_z \qquad \hat{J}_+\hat{J}_- = \hat{J}^2 - \hat{J}_z^2 + \hbar\hat{J}_z$$

Assume j is max m: $\hat{J}_+|\lambda, j\rangle = 0$

Assume j' is min m: $\hat{J}_-|\lambda, j'\rangle = 0$

$$\hat{J}_-\hat{J}_+|\lambda, j\rangle = (\hat{J}^2 - \hat{J}_z^2 - \hbar\hat{J}_z)|\lambda, j\rangle = (\lambda\hbar^2 - j^2\hbar^2 - j\hbar^2)|\lambda, j\rangle = (\lambda - j^2 - j)\hbar^2|\lambda, j\rangle = 0$$

$$\hat{J}_+\hat{J}_-|\lambda, j'\rangle = (\hat{J}^2 - \hat{J}_z^2 + \hbar\hat{J}_z)|\lambda, j'\rangle = (\lambda\hbar^2 - j'^2\hbar^2 + j'\hbar^2)|\lambda, j'\rangle = (\lambda - j'^2 + j')\hbar^2|\lambda, j'\rangle = 0$$

$$\lambda - j^2 - j = 0 \qquad \lambda - j'^2 + j' = 0$$

$\lambda - j^2 - j = 0$

$\lambda - j'^2 + j' = 0$

$\lambda = j^2 + j = j'^2 - j'$

1) $j' = j + 1$

2) $j' = -j$

Max m is j

Min m is $-j$

$\lambda = j^2 + j = j(j+1)$

$m = -j, -j+1, -j+2, \ldots, j-1, j$

$j = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$

$$j = 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, 2, \ldots$$

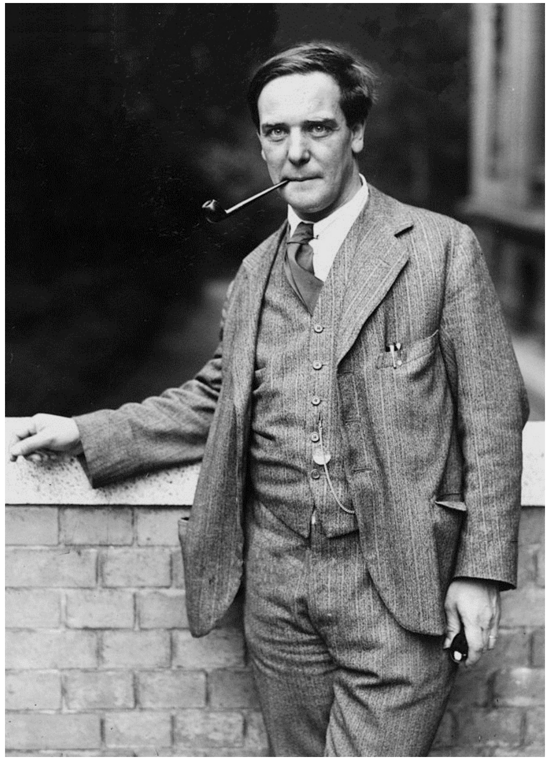
$$j = 0, 1, 2, 3 \ldots$$

Bosons

$$j = \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2}, \ldots$$

Fermions

SUPERFLUIDITY

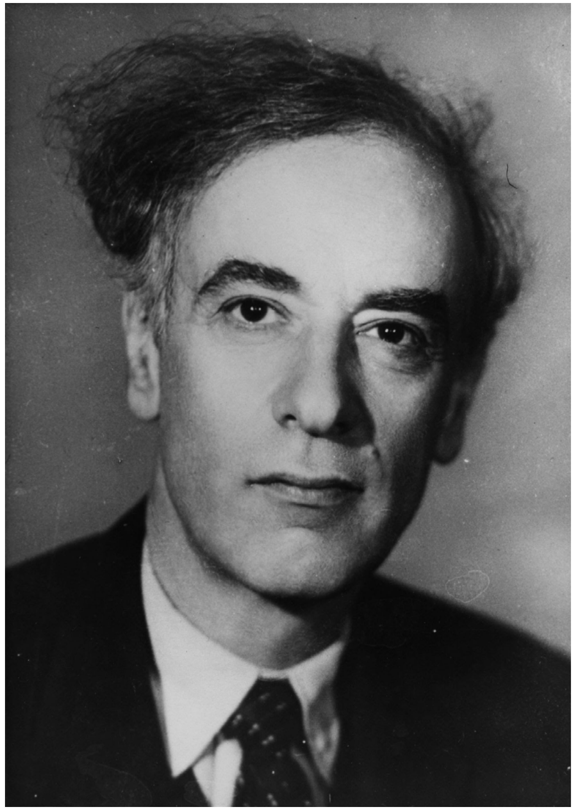


Pyotr Kapitsa

1978

John Frank (Jack) Allen

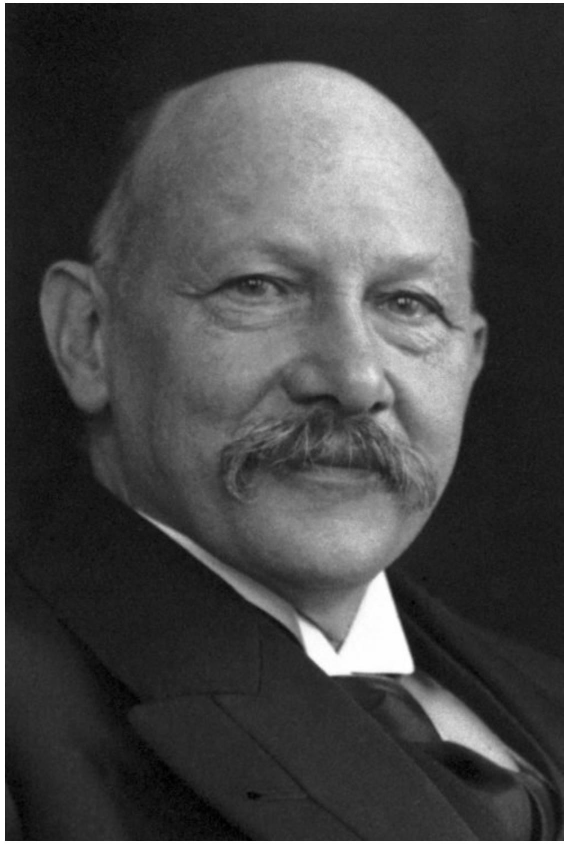
Don Misener



Lev Landau

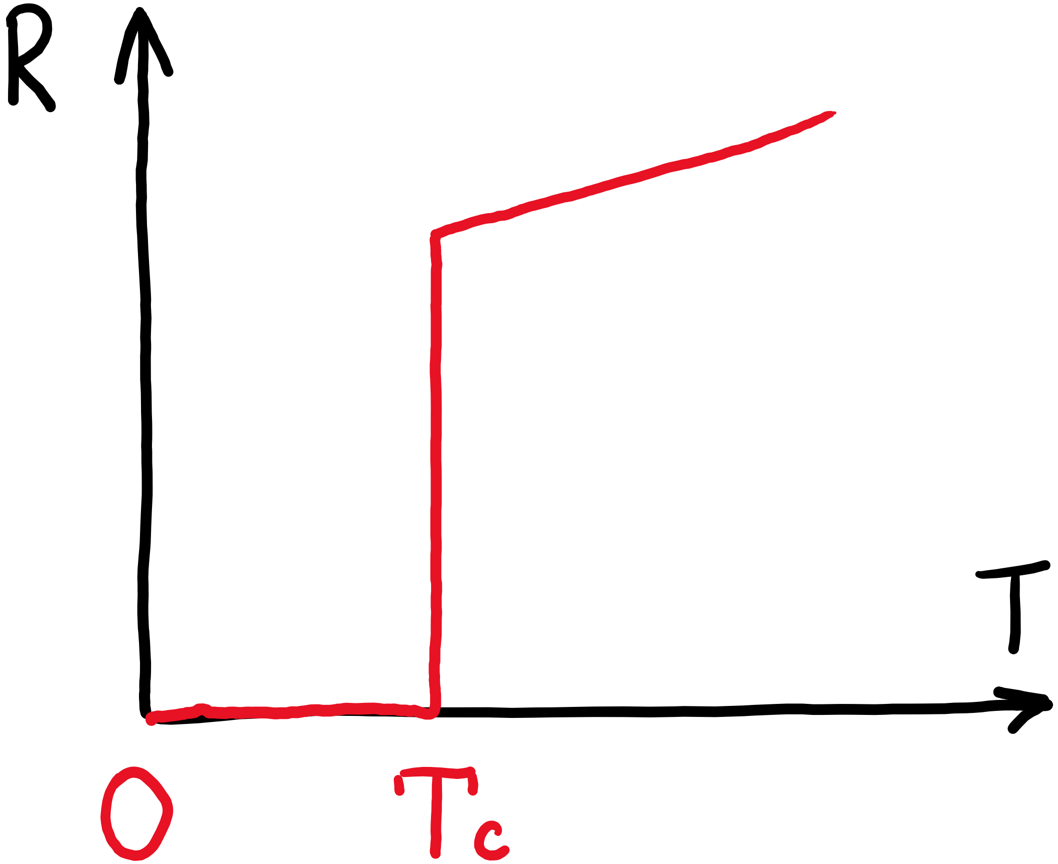
1962

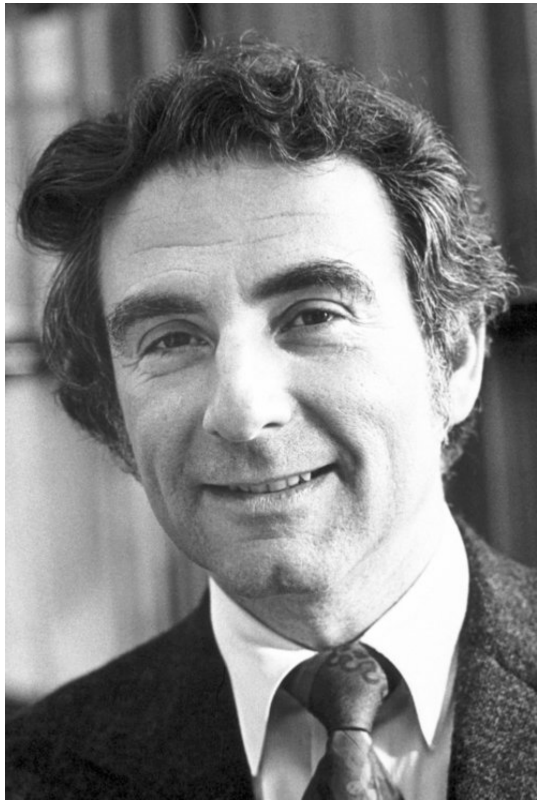
SUPERCONDUCTIVITY



Heike Kamerlingh Onnes

1913





Leon Cooper

1972
with John Bardeen and
John Robert Schrieffer