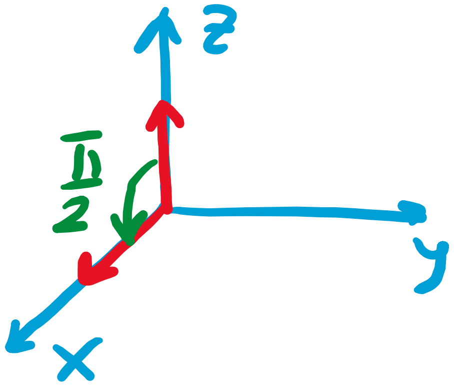
Rotation Operator

$$|+x\rangle = \hat{R}\left(\frac{\pi}{2}\vec{j}\right)|+z\rangle$$

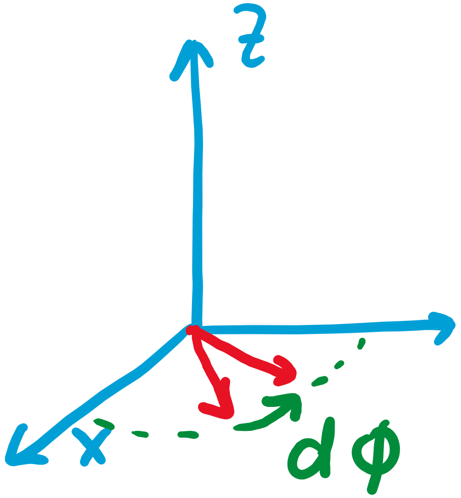


$$\langle +x| = \langle +z|\hat{R}^\dagger\left(\frac{\pi}{2}\vec{j}\right)$$

$$\langle +x|+x\rangle = \langle +z|\underbrace{\hat{R}^\dagger\left(\frac{\pi}{2}\vec{j}\right)\hat{R}\left(\frac{\pi}{2}\vec{j}\right)}_{\hat{1}}|+z\rangle = \langle +z|+z\rangle$$

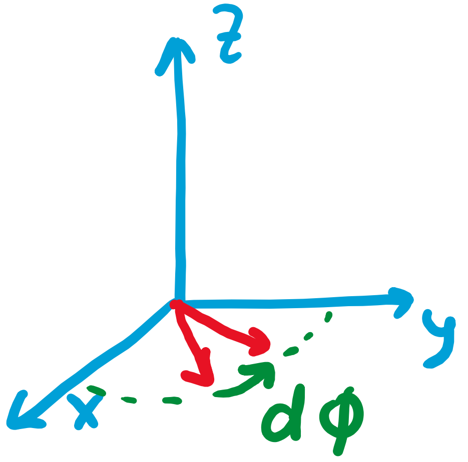
Unitary: $\hat{A}^\dagger\hat{A} = \hat{1}$

$$\hat{R}(d\phi\,\vec{k}) = 1 - \frac{i}{\hbar}\hat{J}_z\,d\phi$$



$\hat{J}_z^\dagger = \hat{J}_z$ Hermitian Operator: $\hat{A}^\dagger = \hat{A}$

$$\hat{R}(d\phi\,\vec{k}) = 1 - \frac{i}{\hbar}\hat{J}_z\, d\phi$$



$$\lim_{N\to\infty}\left(1+\frac{x}{N}\right)^N = e^x$$

$$d\phi = \lim_{N\to\infty}\frac{\phi}{N}$$

$$\hat{R}(\phi k) = \lim_{N\to\infty}\left(1 \overbrace{- \frac{i}{\hbar}\hat{J}_z\frac{\phi}{N}}^{x}\right)^N = e^{-\frac{i\hat{J}_z\phi}{\hbar}}$$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots$$

$$e^{-\frac{i\hat{J}_z\phi}{\hbar}} = 1 + \left(-\frac{i\hat{J}_z\phi}{\hbar}\right) + \frac{1}{2!}\left(-\frac{i\hat{J}_z\phi}{\hbar}\right)^2 + \frac{1}{3!}\left(-\frac{i\hat{J}_z\phi}{\hbar}\right)^3 + \dots$$

$$\hat{R}(\varphi\vec{k})|+z\rangle = \text{const}\cdot|+z\rangle$$

Eigenvector

$$\hat{A}|a\rangle = a|a\rangle$$

Eigenvalue

$$\hat{R}(\varphi\vec{k})|+z\rangle = e^{-\frac{i\hat{J}_z\varphi}{\hbar}}|+z\rangle = \left(1+\left(-\frac{i\hat{J}_z\varphi}{\hbar}\right)+\frac{1}{2!}\left(-\frac{i\hat{J}_z\varphi}{\hbar}\right)^2+\frac{1}{3!}\left(-\frac{i\hat{J}_z\varphi}{\hbar}\right)^3+\dots\right)|+z\rangle$$

$$\hat{J}_z|+z\rangle = \text{const}\cdot|+z\rangle$$

$$\hat{J}_z|+z\rangle = \frac{\hbar}{2}|+z\rangle$$

$$\hat{J}_z|-z\rangle = -\frac{\hbar}{2}|-z\rangle$$

$$\hat{R}(\varphi\vec{k})|+z\rangle = \left(1+\left(-\frac{i\hat{J}_z\varphi}{\hbar}\right)+\frac{1}{2!}\left(-\frac{i\hat{J}_z\varphi}{\hbar}\right)^2+\frac{1}{3!}\left(-\frac{i\hat{J}_z\varphi}{\hbar}\right)^3+\dots\right)|+z\rangle = \left(1+\left(-\frac{i\varphi}{2}\right)+\frac{1}{2!}\left(-\frac{i\varphi}{2}\right)^2+\frac{1}{3!}\left(-\frac{i\varphi}{2}\right)^3+\dots\right)|+z\rangle = e^{-\frac{i\varphi}{2}}|+z\rangle$$

$$\hat{R}(\varphi\vec{k})|-z\rangle = \left(1+\left(-\frac{i\hat{J}_z\varphi}{\hbar}\right)+\frac{1}{2!}\left(-\frac{i\hat{J}_z\varphi}{\hbar}\right)^2+\frac{1}{3!}\left(-\frac{i\hat{J}_z\varphi}{\hbar}\right)^3+\dots\right)|-z\rangle = \left(1+\left(\frac{i\varphi}{2}\right)+\frac{1}{2!}\left(\frac{i\varphi}{2}\right)^2+\frac{1}{3!}\left(\frac{i\varphi}{2}\right)^3+\dots\right)|-z\rangle = e^{\frac{i\varphi}{2}}|-z\rangle$$

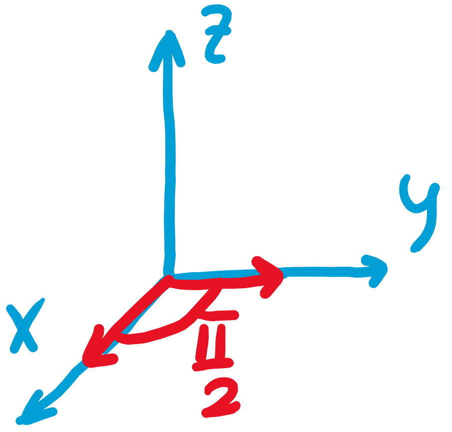
$$\hat{R}(\varphi\vec{k})|+z\rangle = e^{-\frac{i\varphi}{2}}|+z\rangle \qquad \hat{J}_z|+z\rangle = \frac{\hbar}{2}|+z\rangle$$

$$\hat{R}(\varphi\vec{k})|-z\rangle = e^{\frac{i\varphi}{2}}|-z\rangle \qquad \hat{J}_z|-z\rangle = -\frac{\hbar}{2}|-z\rangle$$

$$|+x\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$$

$$\hat{R}(\varphi\vec{k})|+x\rangle = \frac{1}{\sqrt{2}}\hat{R}(\varphi\vec{k})|+z\rangle + \frac{1}{\sqrt{2}}\hat{R}(\varphi k)|-z\rangle = \frac{1}{\sqrt{2}}e^{-\frac{i\varphi}{2}}|+z\rangle + \frac{1}{\sqrt{2}}e^{\frac{i\varphi}{2}}|-z\rangle = e^{-\frac{i\varphi}{2}}\left(\frac{1}{\sqrt{2}}|+z\rangle + \frac{e^{i\varphi}}{\sqrt{2}}|-z\rangle\right)$$

$$\hat{R}(\tfrac{\pi}{2}\vec{k})|+x\rangle = e^{-\frac{i\pi}{4}}\left(\frac{1}{\sqrt{2}}|+z\rangle + \frac{e^{\frac{i\pi}{2}}}{\sqrt{2}}|-z\rangle\right) = e^{-\frac{i\pi}{4}}|+y\rangle$$



$$\hat{R}(\pi\hat{k})|+x\rangle = e^{-\frac{i\pi}{2}}\left(\frac{1}{\sqrt{2}}|+z\rangle + \frac{e^{i\pi}}{\sqrt{2}}|+z\rangle\right) = e^{-\frac{i\pi}{2}}|-x\rangle$$

$$\hat{R}(2\pi\vec{k})|+x\rangle = e^{-i\pi}\left(\frac{1}{\sqrt{2}}|+z\rangle + \frac{e^{i2\pi}}{\sqrt{2}}|-z\rangle\right) = (-1)\left(\frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle\right) = -|+x\rangle$$

$$\hat{R}(d\varphi\,\vec{k}) = 1 - \frac{i}{\hbar}\hat{J}_z\,d\varphi$$

$$\hat{J}_z|+z\rangle = \frac{\hbar}{2}|+z\rangle$$

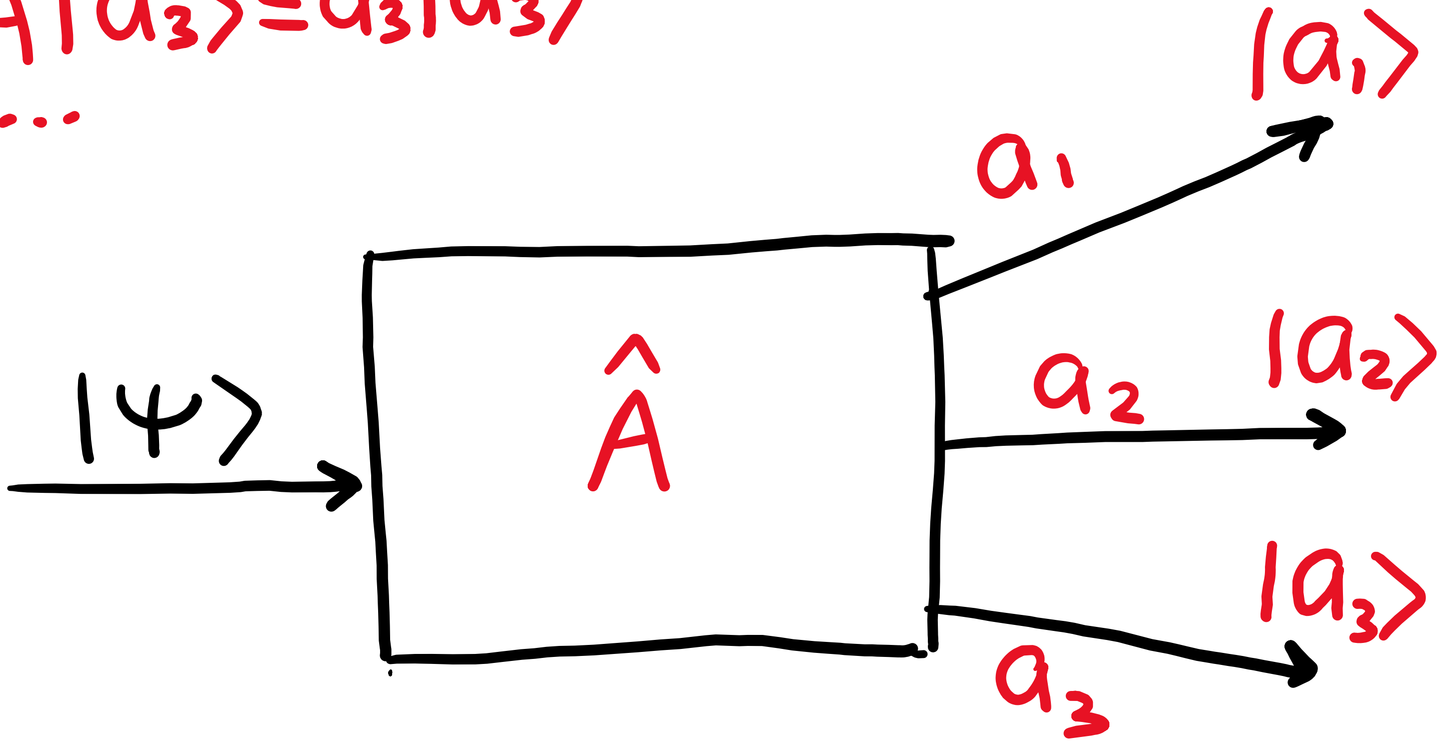
$$\hat{J}_z|-z\rangle = -\frac{\hbar}{2}|-z\rangle$$

$$\hat{A}|a_1\rangle = a_1|a_1\rangle$$

$$\hat{A}|a_2\rangle = a_2|a_2\rangle$$

$$\hat{A}|a_3\rangle = a_3|a_3\rangle$$

$\dots$



$$P = |\langle a_1|\psi\rangle|^2$$

$$P = |\langle a_2|\psi\rangle|^2$$

$$P = |\langle a_3|\psi\rangle|^2$$

$$|\psi\rangle = |+z\rangle\langle +z|\psi\rangle + |-z\rangle\langle -z|\psi\rangle = \underbrace{\Big(|+z\rangle\langle +z| + |-z\rangle\langle -z|\Big)}_{\hat{1}}|\psi\rangle$$

$$|+z\rangle\langle +z| = \hat{P}_+$$

$$\hat{P}_+ + \hat{P}_- = \hat{1}$$

$$|-z\rangle\langle -z| = \hat{P}_-$$

$$|\psi\rangle = |+x\rangle\langle +x|\psi\rangle + |-x\rangle\langle -x|\psi\rangle = \underbrace{\Big(|+x\rangle\langle +x| + |-x\rangle\langle -x|\Big)}_{\hat{1}}|\psi\rangle$$

$$\hat{A}|\psi\rangle = |\varphi\rangle \qquad \hat{A}\big(|+z\rangle\langle+z| + |-z\rangle\langle-z|\big)|\psi\rangle = \big(|+z\rangle\langle+z| + |-z\rangle\langle-z|\big)|\varphi\rangle$$

$$\hat{A}\big(|+z\rangle\langle+z|\psi\rangle + |-z\rangle\langle-z|\psi\rangle\big) = |+z\rangle\langle+z|\varphi\rangle + |-z\rangle\langle-z|\varphi\rangle$$

$$\langle+z|\hat{A}|+z\rangle\langle+z|\psi\rangle + \langle+z|\hat{A}|-z\rangle\langle-z|\psi\rangle = \langle+z|+z\rangle\langle+z|\varphi\rangle + \langle+z|-z\rangle\langle-z|\varphi\rangle$$

$$\langle-z|\hat{A}|+z\rangle\langle+z|\psi\rangle + \langle-z|\hat{A}|-z\rangle\langle-z|\psi\rangle = \langle-z|+z\rangle\langle+z|\varphi\rangle + \langle-z|-z\rangle\langle-z|\varphi\rangle$$

$$\underbrace{\begin{pmatrix} \langle+z|\hat{A}|+z\rangle & \langle+z|\hat{A}|-z\rangle \\ \langle-z|\hat{A}|+z\rangle & \langle-z|\hat{A}|-z\rangle \end{pmatrix}}_{\hat{A}\text{ in } S_z \text{ basis}} \underbrace{\begin{pmatrix} \langle+z|\psi\rangle \\ \langle-z|\psi\rangle \end{pmatrix}}_{|\psi\rangle \text{ in } S_z \text{ basis}} = \underbrace{\begin{pmatrix} \langle+z|\varphi\rangle \\ \langle-z|\varphi\rangle \end{pmatrix}}_{|\varphi\rangle \text{ in } S_z \text{ basis}}$$

$$\hat{A} \xrightarrow[S_z\ basis]{} \begin{pmatrix} \langle +z|\hat{A}|+z\rangle & \langle +z|\hat{A}|-z\rangle \\ \langle -z|\hat{A}|+z\rangle & \langle -z|\hat{A}|-z\rangle \end{pmatrix}$$

$$\hat{1} \xrightarrow[S_z\ basis]{} \begin{pmatrix} \langle +z|\hat{1}|+z\rangle & \langle +z|\hat{1}|-z\rangle \\ \langle -z|\hat{1}|+z\rangle & \langle -z|\hat{1}|-z\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\hat{J}_z \xrightarrow[S_z\ basis]{} \begin{pmatrix} \langle +z|\hat{J}_z|+z\rangle & \langle +z|\hat{J}_z|-z\rangle \\ \langle -z|\hat{J}_z|+z\rangle & \langle -z|\hat{J}_z|-z\rangle \end{pmatrix} = \begin{pmatrix} \frac{\hbar}{2} & 0 \\ 0 & -\frac{\hbar}{2} \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$