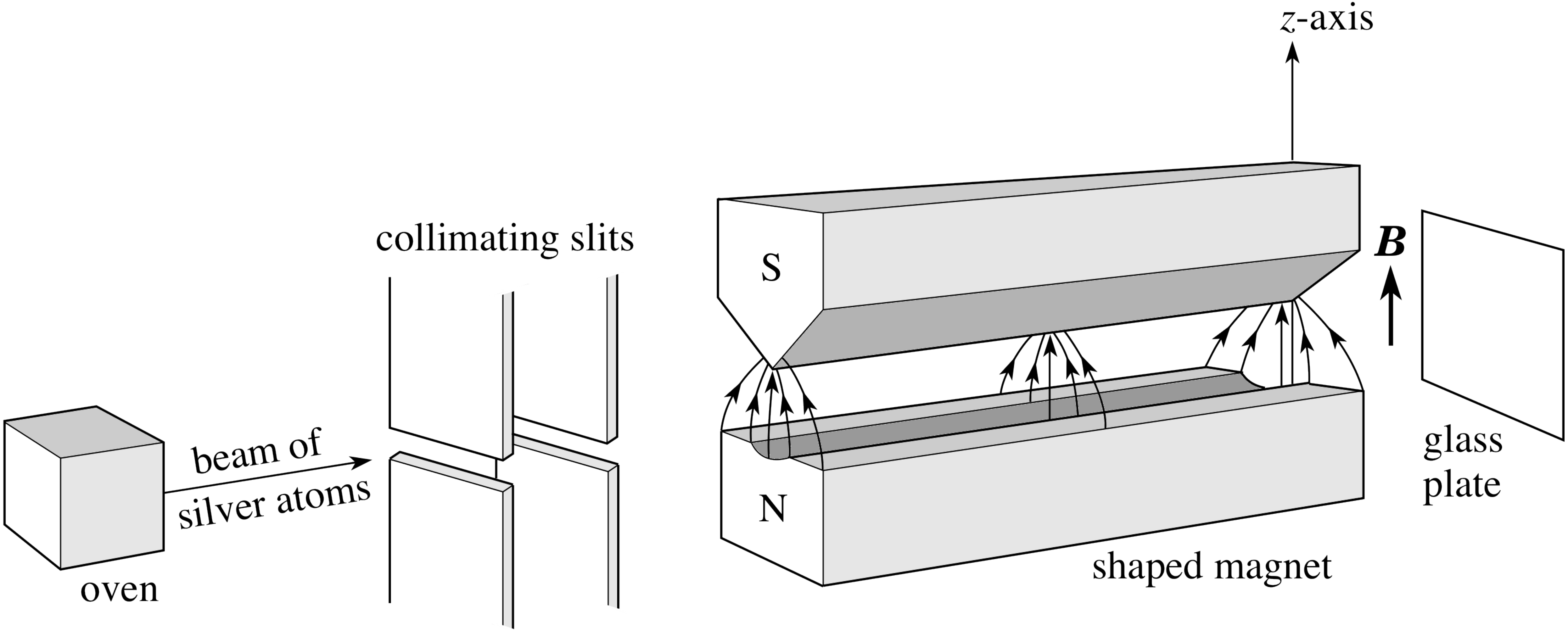
Magnetic dipole in inhomogeneous magnetic field

Energy of interaction: $U = -\vec{\mu} \cdot \vec{B}$

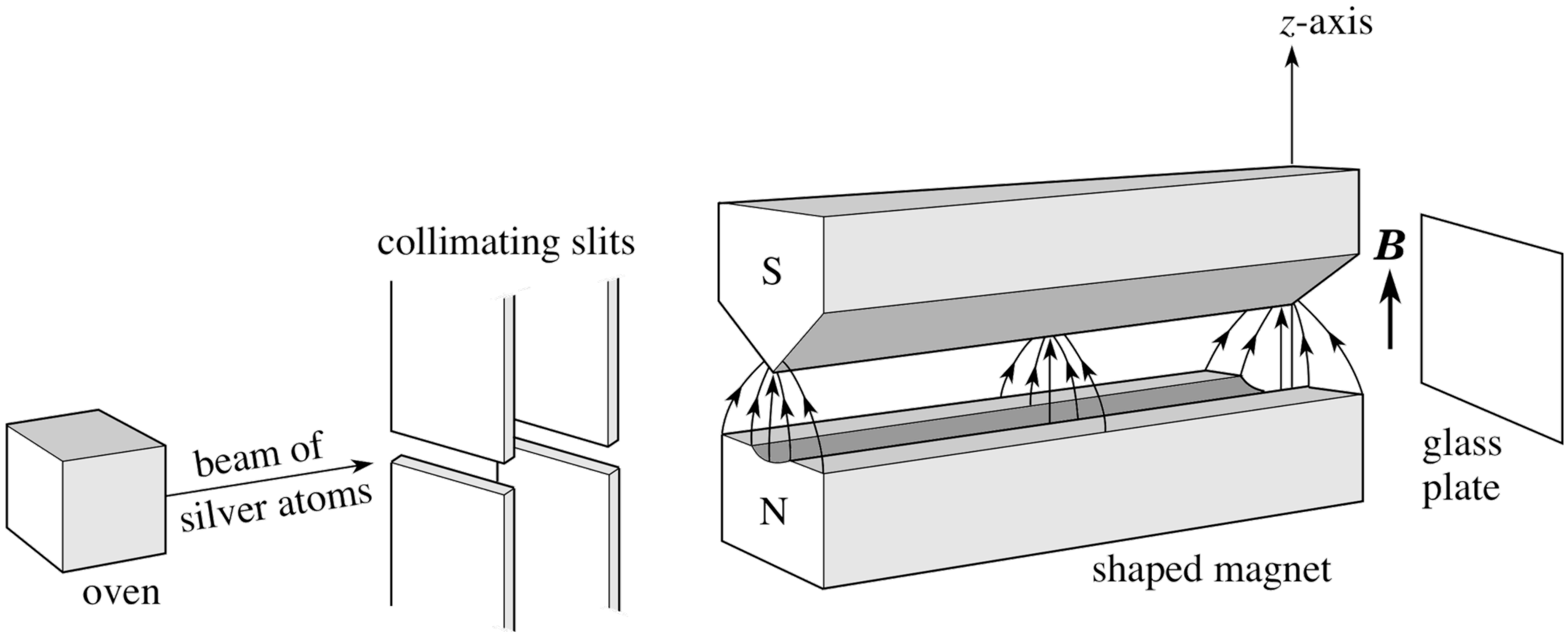
Force: $F = -\nabla U = \nabla(\vec{\mu} \cdot \vec{B})$

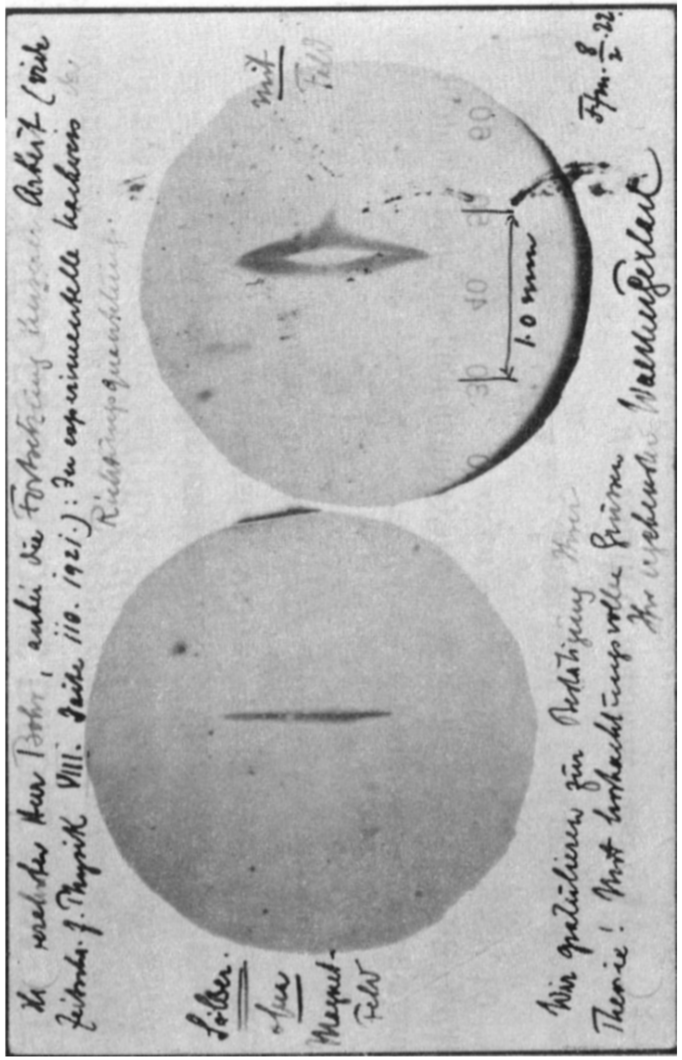
Assume $\frac{\partial B_z}{\partial z}$ is large

$$F_z = \mu_z \cdot \frac{\partial B_z}{\partial z}$$



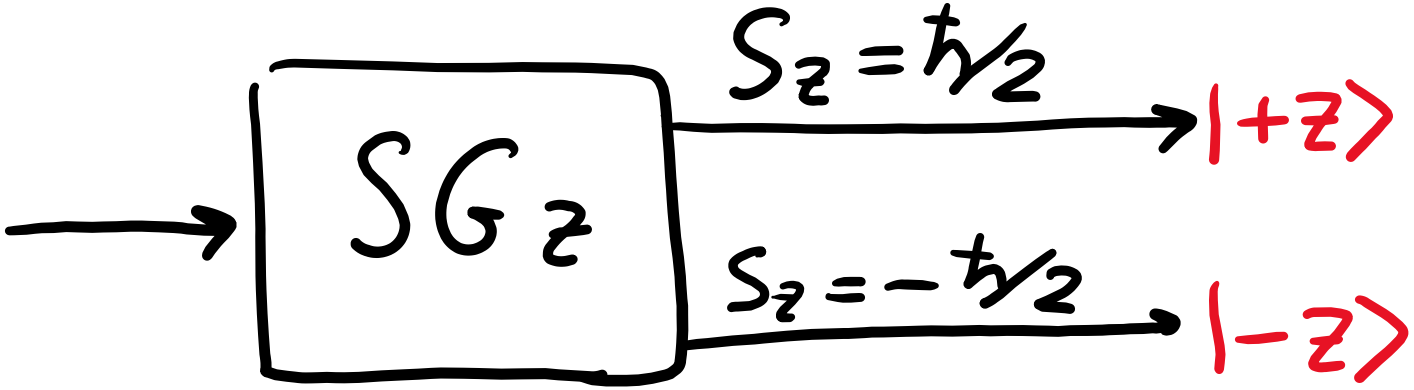
$$F_z = \mu_z \cdot \frac{\partial B_z}{\partial z}$$



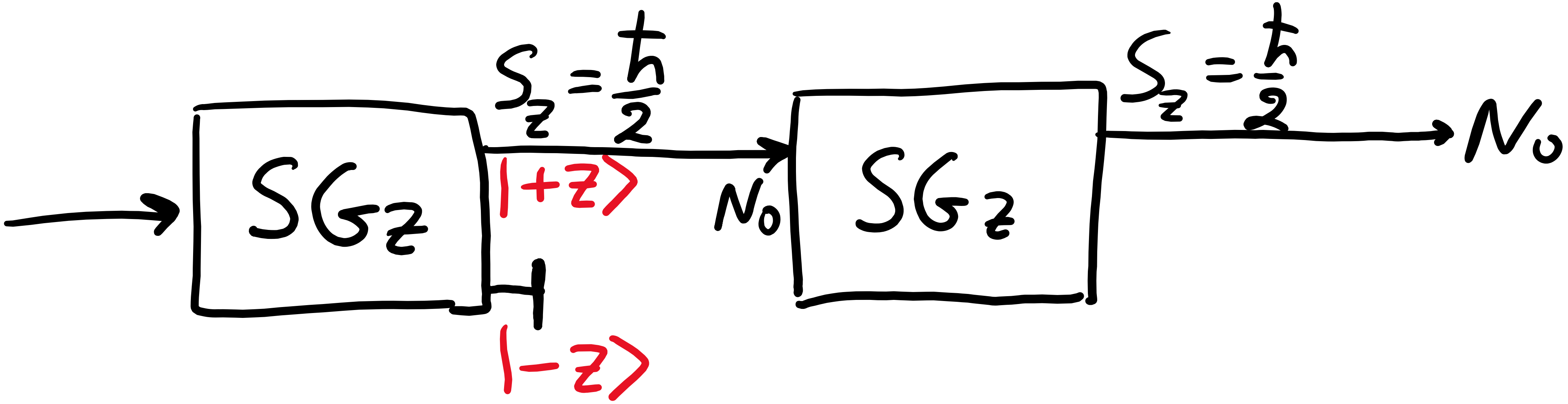


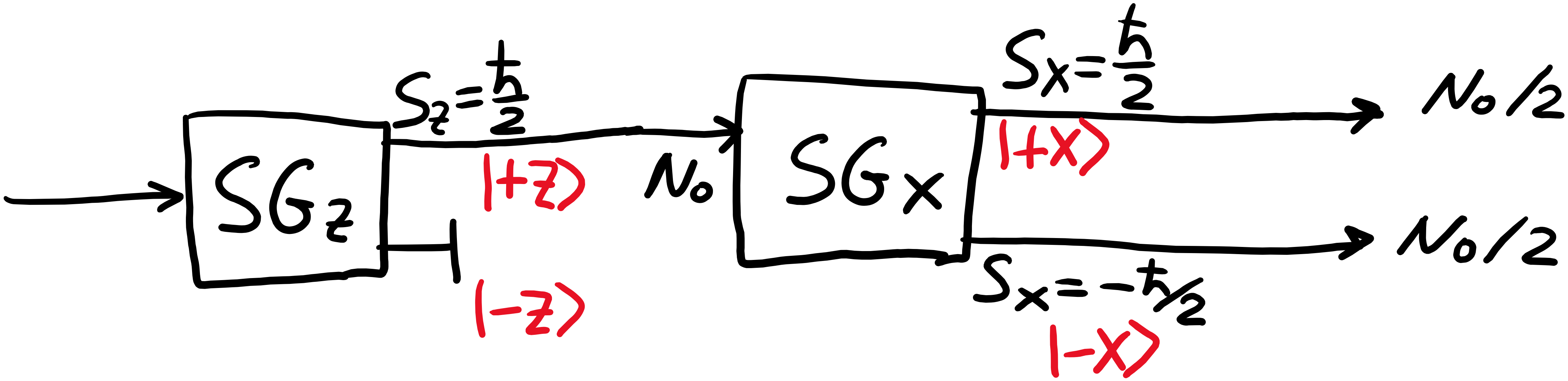
$$\boxed{S_z = \pm \frac{\hbar}{2}}$$

$$\hbar = \frac{h}{2\pi} = 1.055 \cdot 10^{-34}\, J \cdot s$$

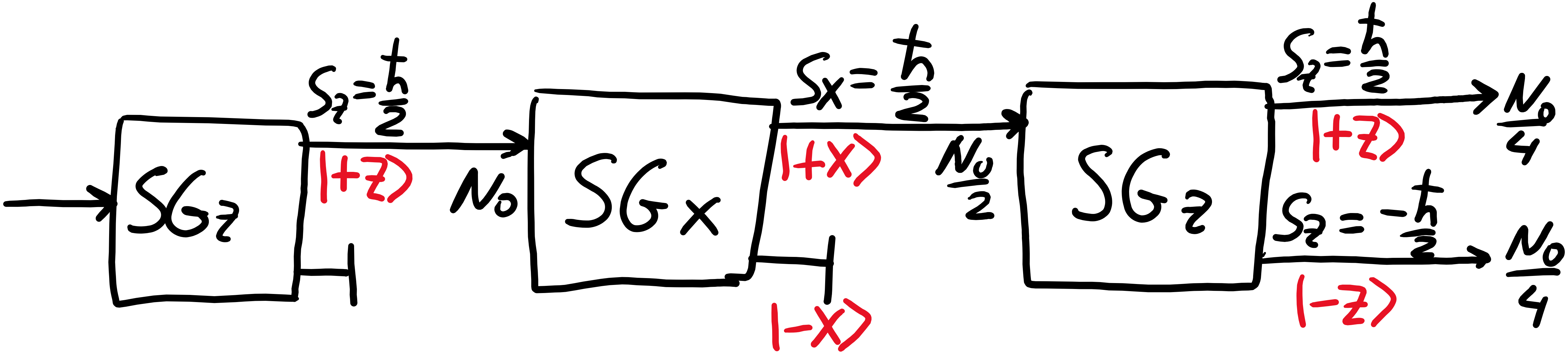

$S_z = \hbar/2$
$|+z\rangle$
SGz
$S_z = -\hbar/2$
$|-z\rangle$

Ket vector

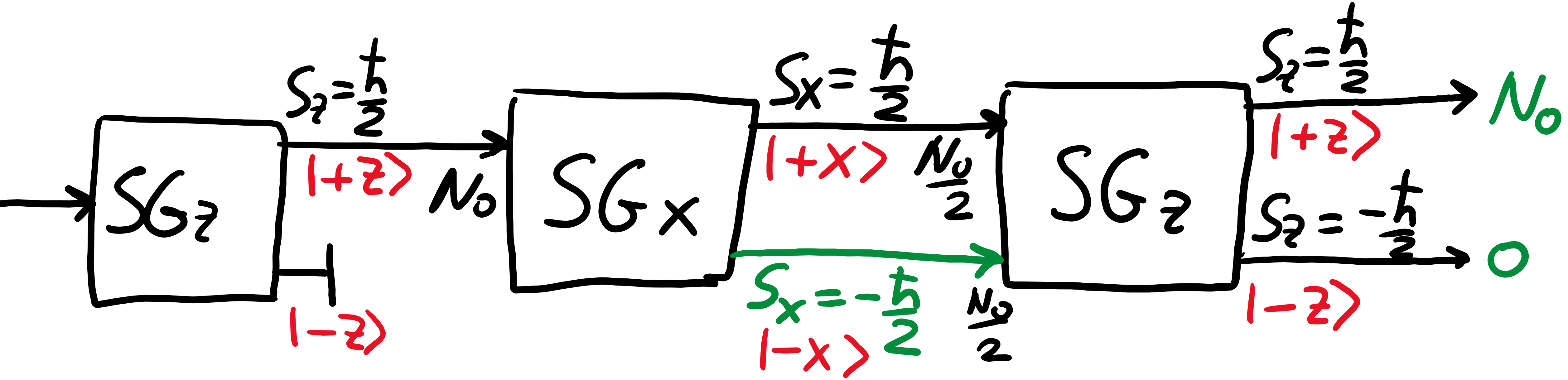

SGz
$S_z = \frac{\hbar}{2}$
$|+z\rangle$
N_0
SGz
$S_z = \frac{\hbar}{2}$
N_0
$|-z\rangle$



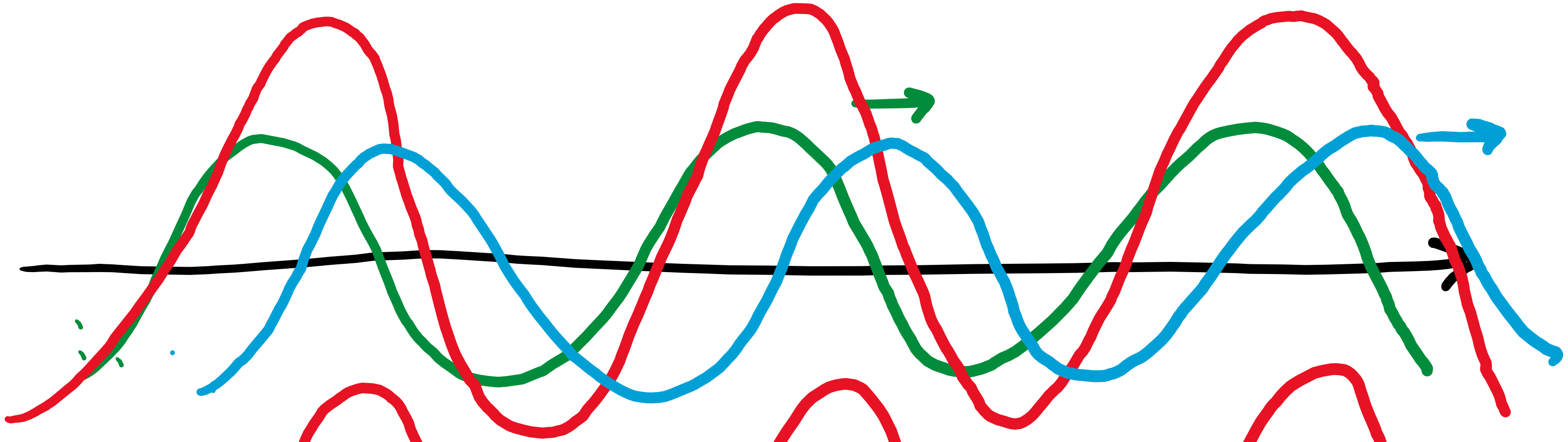
SG_z
$S_z=\frac{\hbar}{2}$
$|+z\rangle$
$|-z\rangle$
N_0
SG_x
$S_x=\frac{\hbar}{2}$
$|+x\rangle$
$N_0/2$
$N_0/2$
$S_x=-\hbar/2$
$|-x\rangle$



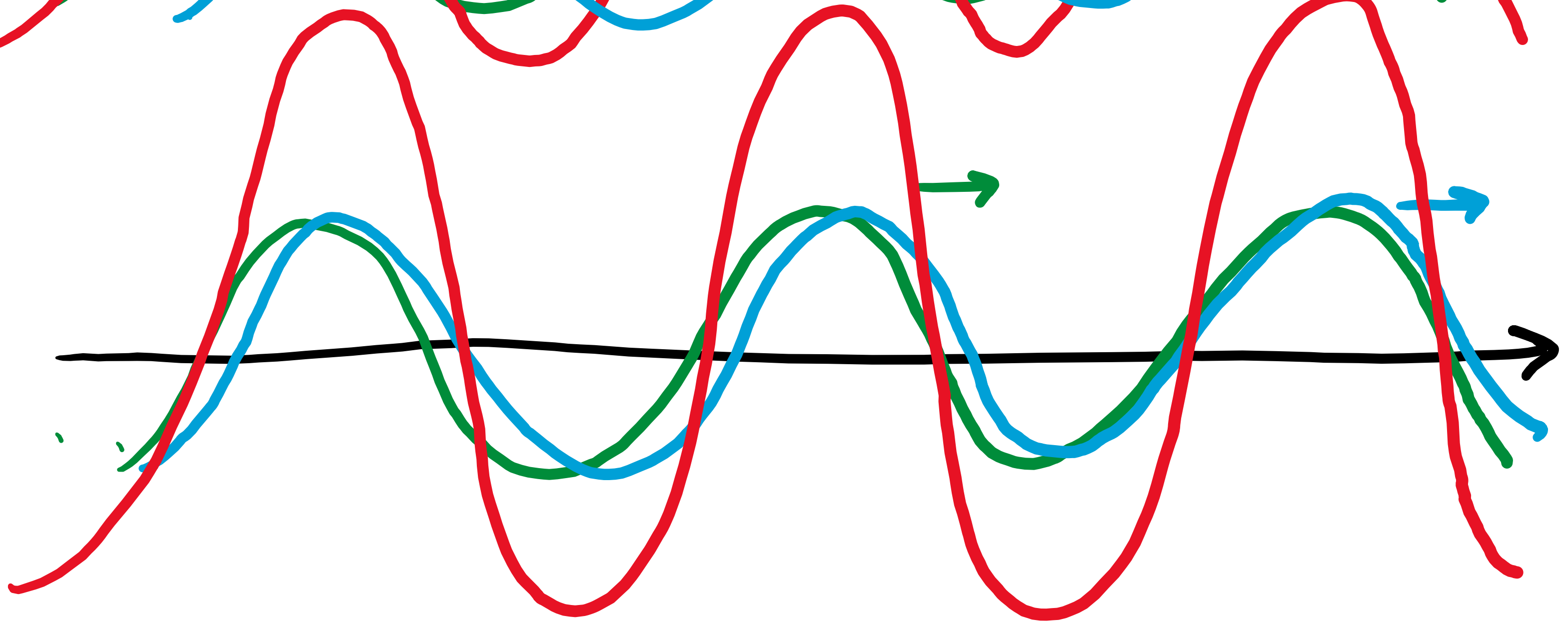
SG_z
$S_z=\frac{\hbar}{2}$
$|+z\rangle$
N_0
SG_x
$S_x=\frac{\hbar}{2}$
$|+x\rangle$
$|-x\rangle$
$\frac{N_0}{2}$
SG_z
$S_z=\frac{\hbar}{2}$
$|+z\rangle$
$\frac{N_0}{4}$
$S_z=-\frac{\hbar}{2}$
$|-z\rangle$
$\frac{N_0}{4}$



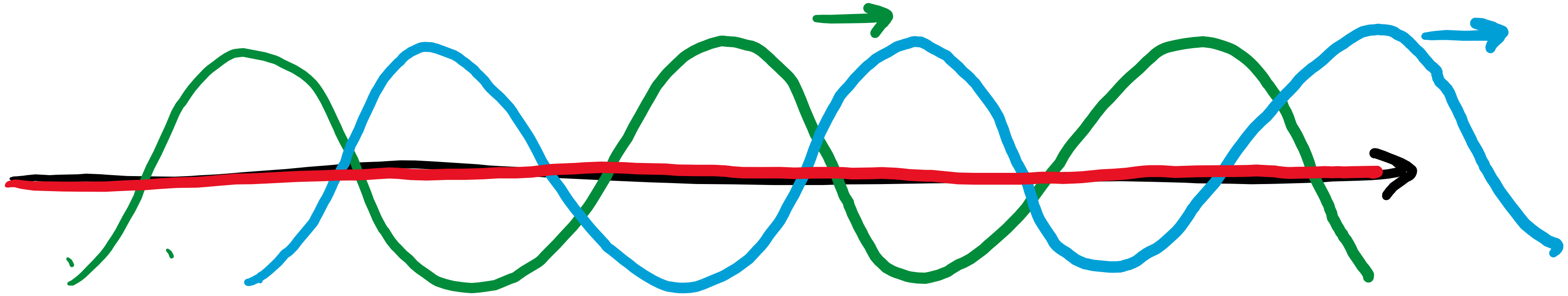
SGz
$S_z = \frac{\hbar}{2}$
$|+z\rangle$
N_0
$|-z\rangle$
SGx
$S_x = \frac{\hbar}{2}$
$|+x\rangle$
$\frac{N_0}{2}$
$S_x = -\frac{\hbar}{2}$
$|-x\rangle$
$\frac{N_0}{2}$
SGz
$S_z = \frac{\hbar}{2}$
$|+z\rangle$
N_0
$S_z = -\frac{\hbar}{2}$
$|-z\rangle$
0



Wave Interference



constructive interference



destructive interference

$I \sim E^2$

$$|\psi\rangle = C_+ |+z\rangle + C_- |-z\rangle$$

Ket vector

State vector

Probability amplitudes

Probability to measure $S_z = \frac{\hbar}{2}$: $|C_+|^2$

Probability to measure $S_z = -\frac{\hbar}{2}$: $|C_-|^2$

Hilbert vector space

How do we multiply vectors?

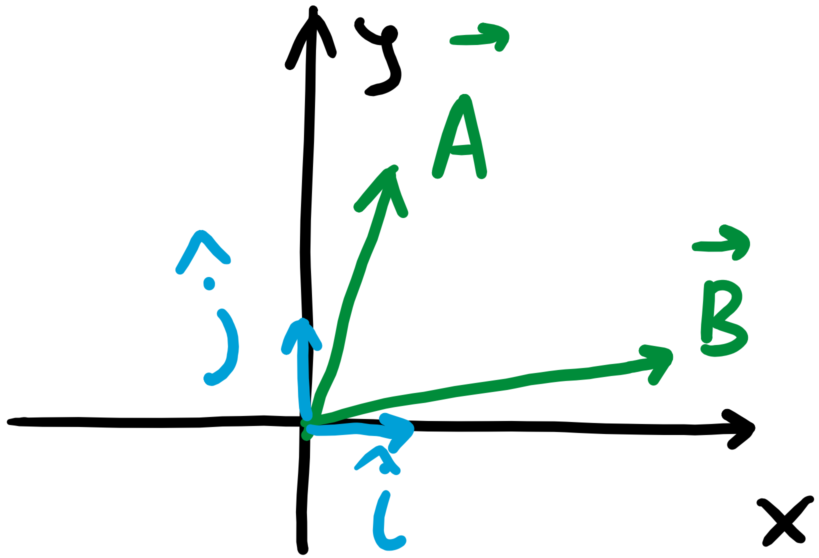
For every ket $|\psi\rangle$ there is a bra vector $\langle\psi|$

$\langle +z|+z\rangle = 1$

$\langle -z|+z\rangle = 0$

$\langle +z|-z\rangle = 0$

$\langle -z|-z\rangle = 1$



$\hat{i}\cdot\hat{i} = 1$

$\hat{i}\cdot\hat{j} = 0$

$\hat{j}\cdot\hat{j} = 1$

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

$\vec{B} = B_x\hat{i} + B_y\hat{j}$

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j})(B_x\hat{i} + B_y\hat{j}) = A_xB_x + A_yB_y$$