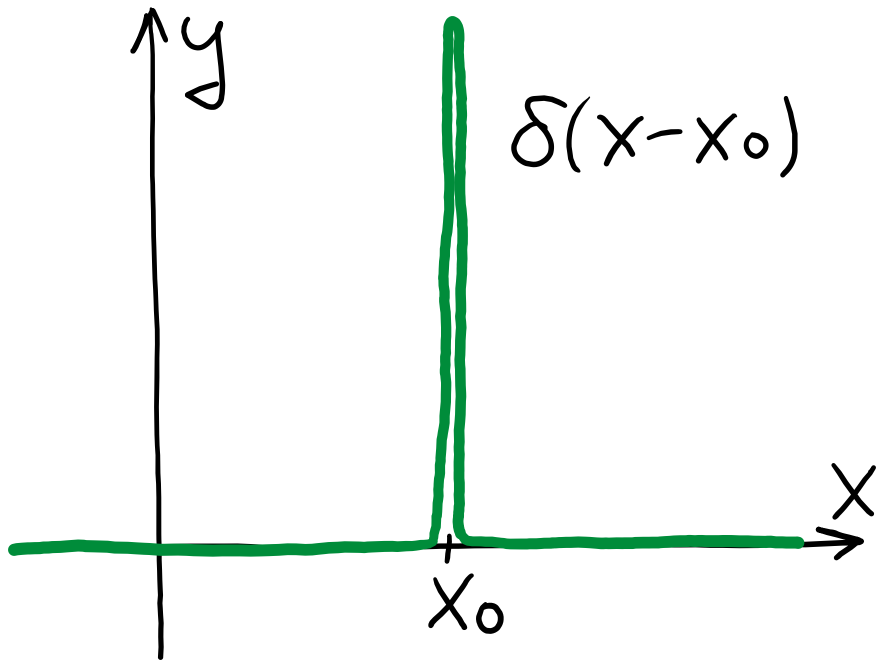
Dirac Delta Function

$$\int_{-\infty}^{\infty} \delta(x-x_0) f(x)\, dx = f(x_0)$$

$$\int_{-\infty}^{\infty} \delta(x)(x^2+2)e^x\, dx = 2$$

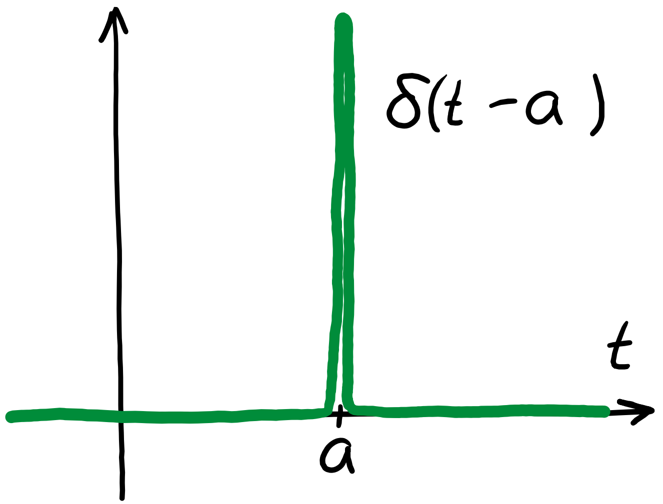
$$f(x) = 1 \implies \int_{-\infty}^{\infty} \delta(x-x_0)\, dx = 1$$

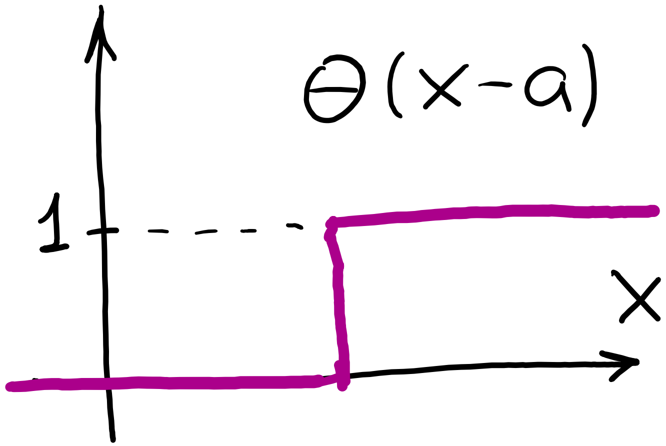


$$\delta(x-x_0) = 0 \quad \text{for } x \neq x_0$$

$$\delta(x) = \delta(-x)$$

$$\int_{-\infty}^{x} \delta(t-a)\,dt = \begin{cases} 0, & x<a \\ 1, & x>a \end{cases} = \Theta(x-a)$$

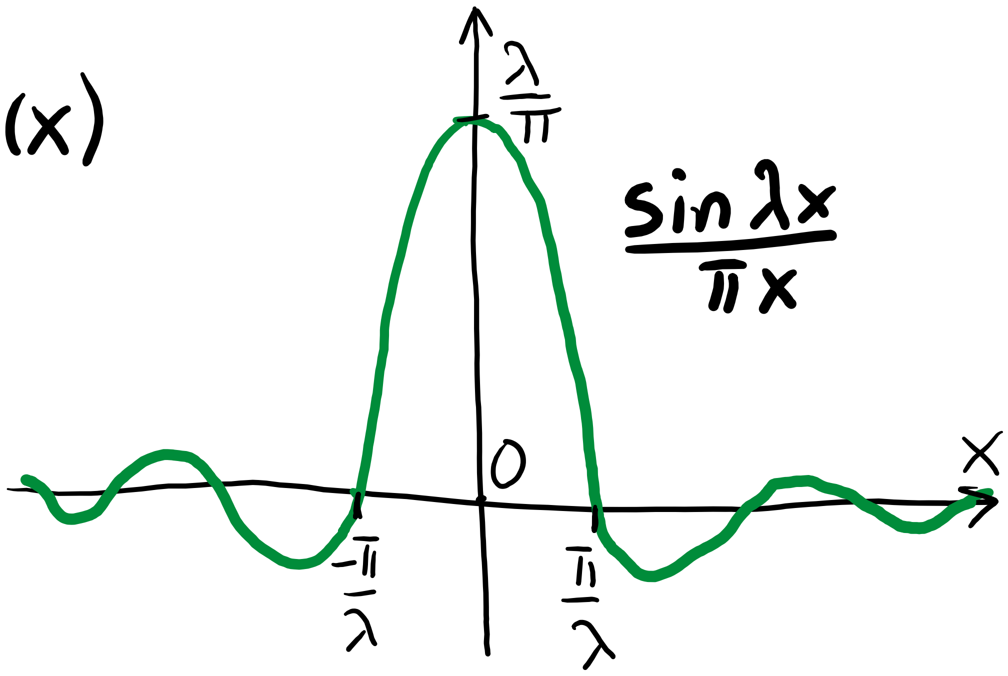




$$\frac{d}{dx}\Theta(x-a) = \delta(x-a)$$

$$\int_{-\infty}^{\infty} \frac{\sin\lambda x}{\pi x}\,dx = 1$$

$$\lim_{\lambda\to\infty} \frac{\sin\lambda x}{\pi x} = \delta(x)$$



$$\int_{-\lambda}^{\lambda} e^{ikx}\,dk = \frac{1}{ix}\left(e^{ix\lambda} - e^{-ix\lambda}\right) = \frac{2}{x}\sin\lambda x$$

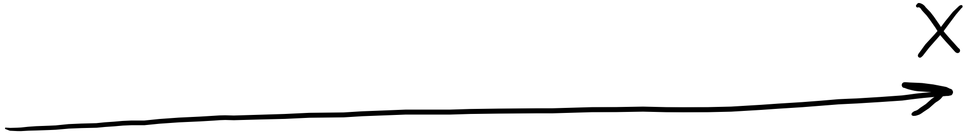
$$\frac{\sin\lambda x}{\pi x} = \frac{1}{2\pi}\int_{-\lambda}^{\lambda} e^{ikx}\,dk$$

$$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}\,dk$$

$$\delta(x) = \lim_{\varepsilon\to 0}\frac{1}{\pi}\frac{\varepsilon}{x^2+\varepsilon^2}$$

$$\delta(x) = \lim_{\alpha\to\infty}\sqrt{\frac{\alpha}{\pi}}\,e^{-\alpha x^2}$$

Position Operator and its eigenstates



$$\hat{X}|x\rangle = x|x\rangle$$

$$|\psi\rangle \overset{?}{=} \sum_i |x_i\rangle\langle x_i|\psi\rangle \qquad \langle\psi| = \sum_j \langle\psi|x_j\rangle\langle x_j|$$

$$\langle\psi|\psi\rangle = \sum_{i,j}\langle\psi|x_j\rangle\langle x_j|x_i\rangle\langle x_i|\psi\rangle = \sum_i \langle\psi|x_i\rangle\langle x_i|\psi\rangle = \sum_i |\langle x_i|\psi\rangle|^2$$

$$|\psi\rangle = \underbrace{\int_{-\infty}^{\infty}|x\rangle\langle x|}_{\hat{1}}\psi\rangle\, dx$$

$$\int_{-\infty}^{\infty}|x\rangle\langle x|\,dx = \hat{1}$$

$$|\psi\rangle = \int_{-\infty}^{\infty} |x\rangle\langle x|\psi\rangle\, dx$$

$$|\psi\rangle = |x'\rangle$$

$$\int_{-\infty}^{\infty} \delta(x-x_0)\, f(x) = f(x_0)$$

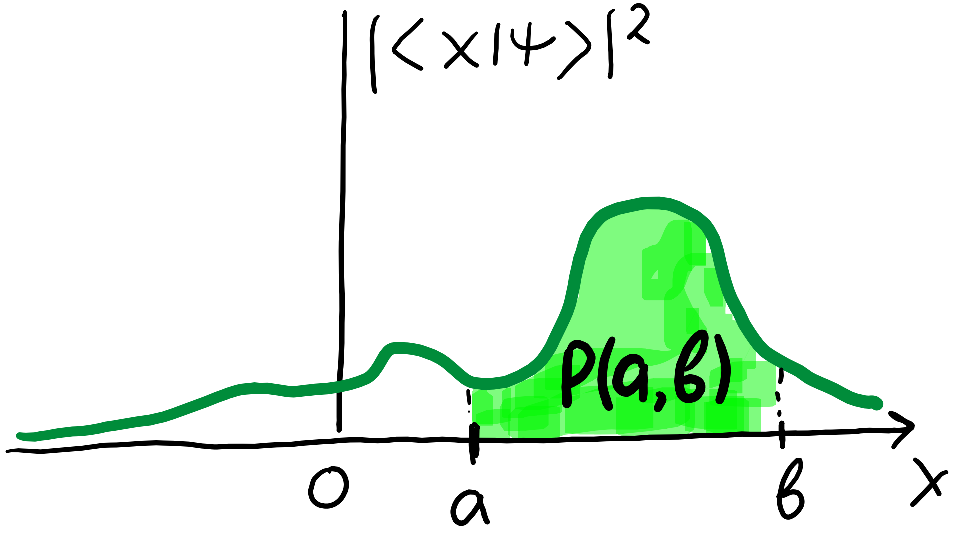
$$|x'\rangle = \int_{-\infty}^{\infty} |x\rangle\underbrace{\langle x|x'\rangle}_{\delta(x-x')} dx$$

$$\langle x|x'\rangle = \delta(x-x')$$

$$\langle\psi| = \int_{-\infty}^{\infty} \langle\psi|x_1\rangle\langle x_1|\, dx_1$$

$$\langle\psi|\psi\rangle = \iint_{-\infty}^{\infty} \langle\psi|x_1\rangle\langle x_1|x\rangle\langle x|\psi\rangle\, dx_1\, dx = \int_{-\infty}^{\infty} \langle\psi|x\rangle\langle x|\psi\rangle\, dx = \int_{-\infty}^{\infty} |\langle x|\psi\rangle|^2 dx = 1$$

$$\int_{-\infty}^{\infty} |\langle x|\psi\rangle|^2 dx = 1$$



Wave function

$$\langle x|\psi\rangle = \psi(x) \qquad \langle\psi|x\rangle = \psi^*(x)$$

$$P(a,b) = \int_a^b |\langle x|\psi\rangle|^2 dx = \int_a^b |\psi(x)|^2 dx$$

$$\langle\varphi|\psi\rangle = \int_{-\infty}^{\infty} \langle\varphi\underbrace{|x\rangle\langle x|}_{1}\psi\rangle\, dx = \int_{-\infty}^{\infty} \varphi^*(x)\,\psi(x)\, dx$$

Translation Operator

$$\hat{T}(a)|x\rangle = |x+a\rangle$$

$$\langle x|\hat{T}^{+}(a) = \langle x+a|$$

$$\langle x|\underbrace{\hat{T}^{+}(a)\hat{T}(a)}_{\hat{1}}|x\rangle = \langle x+a|x+a\rangle$$

$$\hat{T}^{+}(a)\hat{T}(a) = \hat{1}$$