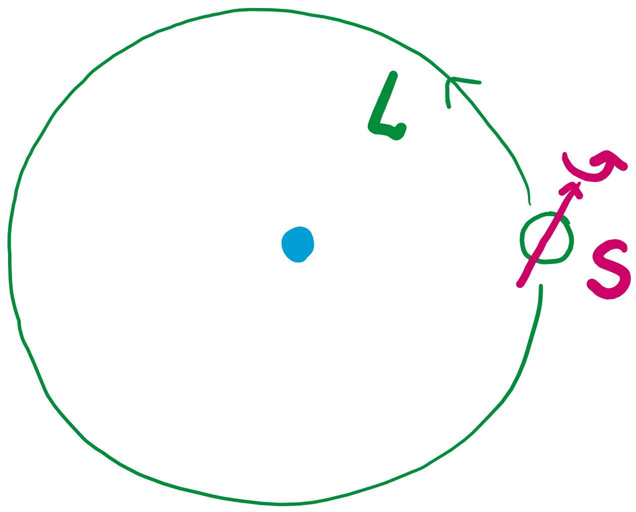
$$\hat{J}^2|j,m\rangle = \hbar^2 j(j+1)|j,m\rangle$$

$$\hat{J}_z|j,m\rangle = \hbar m|j,m\rangle$$

$$\hat{J} \longrightarrow \hat{S} \qquad j \to s$$



$$\hat{S}^2|s,m\rangle = \hbar^2 s(s+1)|s,m\rangle$$

$$\hat{S}_z|s,m\rangle = \hbar m|s,m\rangle$$

$$\vec{\mu} = \frac{q}{2mc}\vec{L}$$

$$\vec{\mu} = \frac{gq}{2mc}\vec{S}$$

$$\hat{S}_+ = \hat{S}_x + i\hat{S}_y \qquad \hat{S}_+|s,m\rangle = \sqrt{s(s+1)-m(m+1)}\,\hbar\,|s,m+1\rangle$$

$$\hat{S}_- = \hat{S}_x - i\hat{S}_y \qquad \hat{S}_-|s,m\rangle = \sqrt{s(s+1)-m(m-1)}\,\hbar\,|s,m-1\rangle$$

$$\hat{S}_+ \xrightarrow[S_z \text{ basis}]{} \begin{pmatrix} \langle +z|\hat{S}_+|+z\rangle & \langle +z|\hat{S}_+|-z\rangle \\ \langle -z|\hat{S}_+|+z\rangle & \langle -z|\hat{S}_+|-z\rangle \end{pmatrix} = \begin{pmatrix} 0 & \hbar \\ 0 & 0 \end{pmatrix} = \hbar \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\hat{S}_+|+z\rangle = 0$$

$$\hat{S}_+|-z\rangle = \hat{S}_+\left|\frac{1}{2}, -\frac{1}{2}\right\rangle = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right) - \left(-\frac{1}{2}\right)\left(-\frac{1}{2}+1\right)}\,\hbar\left|\frac{1}{2}, \frac{1}{2}\right\rangle = \sqrt{\frac{3}{4}+\frac{1}{4}}\,\hbar\left|\frac{1}{2}, \frac{1}{2}\right\rangle = \hbar|+z\rangle$$

$$\langle +z|\hat{S}_+|-z\rangle = \hbar\langle +z|+z\rangle = \hbar$$

$$\hat{S}_- = \hat{S}_+^{\dagger} \quad \Rightarrow \quad \hat{S}_- \xrightarrow[S_z \text{ basis}]{} \hbar \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\hat{S}_+ \xrightarrow[S_z \text{ basis}]{} \hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\hat{S}_+ = \hat{S}_x + i\hat{S}_y$$

$$\hat{S}_- = \hat{S}_x - i\hat{S}_y$$

$$\hat{S}_- \xrightarrow[S_z \text{ basis}]{} \hbar\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\hat{S}_x = \frac{1}{2}(\hat{S}_+ + \hat{S}_-)$$

$$\hat{S}_y = \frac{1}{2i}(\hat{S}_+ - \hat{S}_-)$$

$$\hat{S}_x \xrightarrow[S_z \text{ basis}]{} \frac{1}{2}\left[\hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \hbar\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right] = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\hat{S}_y \xrightarrow[S_z \text{ basis}]{} \frac{1}{2i}\left[\hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} - \hbar\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right] = \frac{\hbar}{2i}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

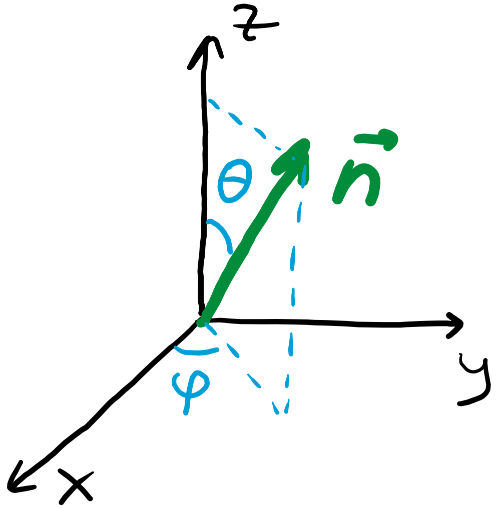
$$\hat{S}_x \xrightarrow[S_z \text{ basis}]{} \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{\hbar}{2}\sigma_x \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\hat{S}_y \xrightarrow[S_z \text{ basis}]{} \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \frac{\hbar}{2}\sigma_y \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\hat{S}_z \xrightarrow[S_z \text{ basis}]{} \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{\hbar}{2}\sigma_z \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{S} \longrightarrow \frac{\hbar}{2}\vec{\sigma} \qquad \hat{S} = S_x \vec{i} + S_y \vec{j} + S_z \vec{K}$$

$$\hat{S}_n = \hat{S} \cdot \vec{n}$$



$$\hat{S}_n \xrightarrow[S_z \text{ basis}]{} \frac{\hbar}{2}\sigma_x \cdot n_x + \frac{\hbar}{2}\sigma_y \cdot n_y + \frac{\hbar}{2}\sigma_z n_z$$

$n_x = \sin\theta\cos\varphi$

$n_y = \sin\theta\sin\varphi$

$n_z = \cos\theta$

$$= \frac{\hbar}{2}\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \sin\theta\cos\varphi + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\sin\theta\sin\varphi + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cos\theta\right)$$

$$= \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta\cos\varphi - i\sin\theta\sin\varphi \\ \sin\theta\cos\varphi + i\sin\theta\sin\varphi & -\cos\theta \end{pmatrix}$$

$$= \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta(\cos\varphi - i\sin\varphi) \\ \sin\theta(\cos\varphi + i\sin\varphi) & -\cos\theta \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta \cdot e^{i\varphi} & -\cos\theta \end{pmatrix}$$

$$\hat{S}_n \xrightarrow[S_z \text{ basis}]{} \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix}$$

$$\det\begin{vmatrix} \cos\theta-\lambda & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta-\lambda \end{vmatrix} = 0$$

$$(\cos\theta-\lambda)(-\cos\theta-\lambda)-\sin^2\theta=0$$

$$\lambda^2-\cos^2\theta-\sin^2\theta=0$$

$$\lambda^2=1$$

$$\lambda=\pm 1$$

$\lambda = 1$ Eigenvalue $\frac{\hbar}{2}$:

$$\begin{pmatrix} \cos\theta-1 & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta-1 \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$X_1(\cos\theta-1)+X_2\sin\theta e^{-i\varphi}=0$$

$$x_1(\cos\theta-1)+x_2\sin\theta e^{-i\varphi}=0$$

$$\cos\theta-1=-2\sin^2\frac{\theta}{2}$$

$$-x_1\cdot 2\sin^2\frac{\theta}{2}+x_2\cdot 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}e^{-i\varphi}=0$$

$$\sin\theta=2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$-x_1\sin\frac{\theta}{2}+x_2\cos\frac{\theta}{2}e^{-i\varphi}=0$$

$$|+n\rangle=\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}e^{i\varphi}\end{pmatrix}=\cos\frac{\theta}{2}|+z\rangle+\sin\frac{\theta}{2}e^{i\varphi}|-z\rangle$$

$S=1 \qquad m=-1,0,1$

$|1,1\rangle \qquad |1,0\rangle \qquad |1,-1\rangle$

$$\vec{S}_z \xrightarrow[S_z\ \text{basis}]{} \begin{pmatrix} \langle 1,1|\hat{S}_z|1,1\rangle & \langle 1,1|\hat{S}_z|1,0\rangle & \langle 1,1|\hat{S}_z|1,-1\rangle \\ \langle 1,0|\hat{S}_z|1,1\rangle & \langle 1,0|\hat{S}_z|1,0\rangle & \langle 1,0|\hat{S}_z|1,-1\rangle \\ \langle 1,-1|\hat{S}_z|1,1\rangle & \langle 1,-1|\hat{S}_z|1,0\rangle & \langle 1,-1|\hat{S}_z|1,-1\rangle \end{pmatrix} = \begin{pmatrix} \hbar & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar \end{pmatrix} = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\vec{S}_+ \xrightarrow[S_z\ \text{basis}]{} \begin{pmatrix} \langle 1,1|\hat{S}_+|1,1\rangle & \langle 1,1|\hat{S}_+|1,0\rangle & \langle 1,1|\hat{S}_+|1,-1\rangle \\ \langle 1,0|\hat{S}_+|1,1\rangle & \langle 1,0|\hat{S}_+|1,0\rangle & \langle 1,0|\hat{S}_+|1,-1\rangle \\ \langle 1,-1|\hat{S}_+|1,1\rangle & \langle 1,-1|\hat{S}_+|1,0\rangle & \langle 1,-1|\hat{S}_+|1,-1\rangle \end{pmatrix}$$

$$\hat{S}_+|1,1\rangle = 0 \qquad \hat{S}_+|1,0\rangle = \sqrt{1(1+1)-0(0+1)}\,\hbar|1,1\rangle = \sqrt{2}\,\hbar|1,1\rangle$$

$$\hat{S}_+|1,-1\rangle = \sqrt{1(1+1)-(-1)(-1+1)}\,\hbar|1,0\rangle = \sqrt{2}\,\hbar|1,0\rangle$$

$$\langle 1,0|\hat{S}_+|1,-1\rangle = \sqrt{2}\,\hbar\langle 1,0|1,0\rangle = \sqrt{2}\,\hbar$$

$$\langle 1,1|\hat{S}_+|1,0\rangle = \sqrt{2}\,\hbar\langle 1,1|1,1\rangle = \sqrt{2}\,\hbar$$

$$\vec{S}_+ \xrightarrow[S_z \text{ basis}]{} \begin{pmatrix} \langle 1,1|\hat{S}_+|1,1\rangle & \langle 1,1|\hat{S}_+|1,0\rangle & \langle 1,1|\hat{S}_+|1,-1\rangle \\ \langle 1,0|\hat{S}_+|1,1\rangle & \langle 1,0|\hat{S}_+|1,0\rangle & \langle 1,0|\hat{S}_+|1,-1\rangle \\ \langle 1,-1|\hat{S}_+|1,1\rangle & \langle 1,-1|\hat{S}_+|1,0\rangle & \langle 1,-1|\hat{S}_+|1,-1\rangle \end{pmatrix} = \hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\vec{S}_- = \hat{S}_+^{+} = \hbar\begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

$$\hat{S}_x = \frac{1}{2}\left(\hat{S}_+ + \hat{S}_-\right)$$

$$\hat{S}_y = \frac{1}{2i}\left(\hat{S}_+ - \hat{S}_-\right)$$

$$\vec{S}_+ \xrightarrow[S_z\text{ basis}]{} \hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\hat{S}_- \xrightarrow[S_z\text{ basis}]{} \hbar\begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

$$\hat{S}_x \xrightarrow[S_z\text{ basis}]{} \frac{\hbar}{2}\left[\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}\right] = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\hat{S}_y \xrightarrow[S_z\text{ basis}]{} \frac{\hbar}{2i}\left[\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}\right] = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$S = 2 \qquad m = -2, -1, 0, 1, 2$

$|2,-2\rangle \qquad |2,-1\rangle \qquad |2,0\rangle \qquad |2,1\rangle \qquad |2,2\rangle$

$$\hat{S}_z \xrightarrow[S_z \text{ basis}]{} \hbar \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$

$$\hat{S}_+ \xrightarrow[S_z \text{ basis}]{} \begin{pmatrix}
\cancel{\langle 2,2|\hat{S}_+|2,2\rangle}\ 0 & \langle 2,2|\hat{S}_+|2,1\rangle\ 2\hbar & \cancel{\langle 2,2|\hat{S}_+|2,0\rangle}\ 0 & \cancel{\langle 2,2|\hat{S}_+|2,-1\rangle}\ 0 & \cancel{\langle 2,2|\hat{S}_+|2,-2\rangle}\ 0 \\
\cancel{\langle 2,1|\hat{S}_+|2,2\rangle}\ 0 & \cancel{\langle 2,1|\hat{S}_+|2,1\rangle}\ 0 & \langle 2,1|\hat{S}_+|2,0\rangle\ \sqrt{6}\hbar & \cancel{\langle 2,1|\hat{S}_+|2,-1\rangle}\ 0 & \cancel{\langle 2,1|\hat{S}_+|2,-2\rangle}\ 0 \\
\cancel{\langle 2,0|\hat{S}_+|2,2\rangle}\ 0 & \cancel{\langle 2,0|\hat{S}_+|2,1\rangle}\ 0 & \cancel{\langle 2,0|\hat{S}_+|2,0\rangle}\ 0 & \langle 2,0|\hat{S}_+|2,-1\rangle\ \sqrt{6}\hbar & \cancel{\langle 2,0|\hat{S}_+|2,-2\rangle}\ 0 \\
\cancel{\langle 2,-1|\hat{S}_+|2,2\rangle}\ 0 & \cancel{\langle 2,-1|\hat{S}_+|2,1\rangle}\ 0 & \cancel{\langle 2,-1|\hat{S}_+|2,0\rangle}\ 0 & \cancel{\langle 2,-1|\hat{S}_+|2,-1\rangle}\ 0 & \langle 2,-1|\hat{S}_+|2,-2\rangle\ 2\hbar \\
\cancel{\langle 2,-2|\hat{S}_+|2,2\rangle}\ 0 & \cancel{\langle 2,-2|\hat{S}_+|2,1\rangle}\ 0 & \cancel{\langle 2,-2|\hat{S}_+|2,0\rangle}\ 0 & \cancel{\langle 2,-2|\hat{S}_+|2,-1\rangle}\ 0 & \cancel{\langle -2,2|\hat{S}_+|2,-2\rangle}\ 0
\end{pmatrix}$$

$$\hat{S}_+|2,1\rangle = \sqrt{\underbrace{2(2+1)}_{6} - \underbrace{1(1+1)}_{2}}\,\hbar\,|2,2\rangle = 2\hbar|2,2\rangle \Rightarrow \langle 2,2|\hat{S}_+|2,1\rangle = 2\hbar$$

$$\hat{S}_+|2,0\rangle = \sqrt{\underbrace{2(2+1)}_{6} - 0(0+1)}\,\hbar\,|2,1\rangle = \sqrt{6}\hbar|2,1\rangle \Rightarrow \langle 2,1|\hat{S}_+|2,0\rangle = \sqrt{6}\hbar$$

$$\hat{S}_+|2,-1\rangle = \sqrt{2(2+1) - (-1)(-1+1)}\,\hbar\,|2,0\rangle = \sqrt{6}\hbar|2,0\rangle \Rightarrow \langle 2,0|\hat{S}_+|2,-1\rangle = \sqrt{6}\hbar$$

$$\hat{S}_+|2,-2\rangle = \sqrt{2(2+1) - (-2)(-2+1)}\,\hbar\,|2,-1\rangle = 2\hbar|2,-1\rangle \Rightarrow \langle 2,-1|\hat{S}_+|2,-2\rangle = 2\hbar$$

$$\hat{S}_+ \xrightarrow[S_z \text{ basis}]{} \hbar \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{6} & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \hat{S}_- \xrightarrow[S_z \text{ basis}]{} \hbar \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{6} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix}$$

$$\hat{S}_x = \frac{1}{2}\left(\hat{S}_+ + \hat{S}_-\right) = \frac{\hbar}{2}\begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & \sqrt{6} & 0 & 0 \\ 0 & \sqrt{6} & 0 & \sqrt{6} & 0 \\ 0 & 0 & \sqrt{6} & 0 & 2 \\ 0 & 0 & 0 & & 0 \end{pmatrix}$$